Multivariate Hawkes Processes - Application to an insurance company

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Abstract

In comparison with everyday products (e.g. food, clothes, movies, etc.), insurance has a specific way of consumption. Subscription to a new insurance product or cover from a customer is often associated with a life event for this customer. For instance, subscribing or modifying a car insurance with new covers often implies a vehicle purchase, which could be explained by a new job (new location and new wage), a marriage or a birth (purchase of a family car), etc. Thus, being able to anticipate such events could be an asset for an insurance company in order to manage their marketing actions and also the relationship with customers. Multivariate Hawkes processes could fit well to model occurrence of these life events, since this class of point process allows interdependence between events. Indeed it is reasonable to believe that the different life events are correlated (e.g. marriage and birth). Besides, this modeling allows including internal and external data related to customers and using state-of-the-art Machine Learning techniques to be able to predict which event is going to occur and when.

Motivation

Predict this kind of events...

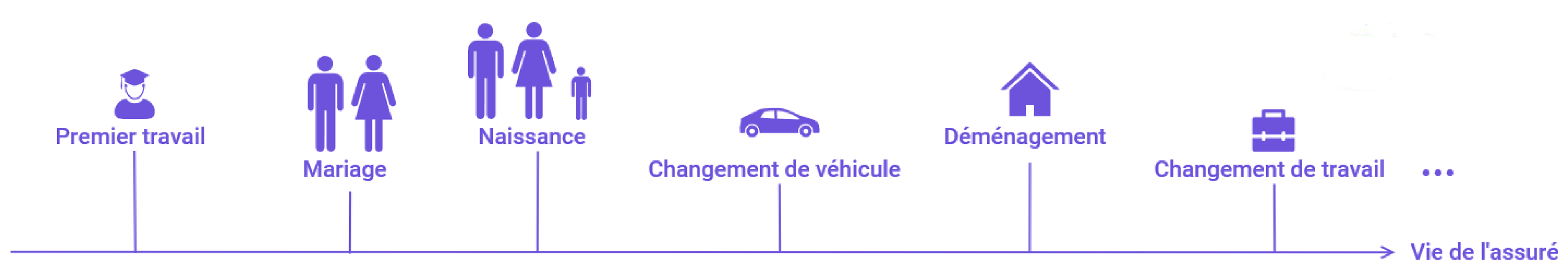

Fig. 1: Overview of motivations for the project

... in order to bring information for three types of useful indicators for our insurance company:

- Estimation of churn rate of customers,
- Recommender system to suggest additional insurance covers,
- Estimation of the "future value" of customers: how much will customers cost in the future?

Problem statement

(Multivariate Hawkes process [3]) Let's consider a list of m counting processes $\{N_1, ..., N_m\}$. We denote $\{t_1^i, .., t_{n(i)}^i\}$ the sequence of past occurrences until time t for the counting process i and $n(i)$ the number of events for process i. $\mathcal{H}(t)$ is the history until time t. The intensity function of $N_i(.)$ is defined by:

$$\lambda_i^*(t) = \lim_{dt \to 0^+} \frac{\mathbb{E}[N_i(t+dt) - N_i(t)|\mathcal{H}(t)]}{dt} \quad (1)$$

In case of multivariate Hawkes process, λ_i^* is of the form

$$\begin{aligned}\lambda_i^*(t) &= \lambda_i + \sum_{j=1}^{m} \int_0^t \mu_{j,i}(t-u) dN_j(u) \\ &= \lambda_i + \sum_{j=1}^{m} \sum_{k=1}^{n(j)} \mu_{j,i}(t - t_k^j)\end{aligned} \quad (2)$$

where λ_i is the *background intensity* and $\mu_{j,i}$ is the *excitation function*.

Main steps of modelling: useful results about Hawkes processes

1) Ensuring that intensity will not explode: expectation of a univariate Hawkes process [2]

From

$$\lambda^*(t) = \lim_{dt \to 0^+} \frac{\mathbb{E}[N(t+dt) - N(t)|\mathcal{H}(t)]}{dt} = \frac{\mathbb{E}[dN(t)|\mathcal{H}(t)]}{dt}$$

we obtain

$$g(t) = \mathbb{E}[\lambda^*(t)] = \frac{\mathbb{E}[\mathbb{E}[dN(t)|\mathcal{H}(t)]]}{dt} = \frac{\mathbb{E}[dN(t)]}{dt}$$

Moreover,

$$g(t) = \mathbb{E}[\lambda^*(t)] = \mathbb{E}\Big[\lambda + \int_0^t \mu(t-u) dN(u)\Big] = \lambda + \int_0^t \mu(t-u)\mathbb{E}[dN(u)]$$

$$g(t) = \lambda + \int_0^t \mu(t-u) g(u) du \quad (3)$$

2) Estimating parameters: maximum of likelihood [4]

$$\begin{aligned}L &= \Big[\prod_{i=1}^{n} f(t_i|\mathcal{H}(t_{i-1}))\Big](1 - F(T|\mathcal{H}(t_n)) \\ &= \Big[\prod_{i=1}^{n} \lambda^*(t_i) \exp\Big(-\int_{t_{i-1}}^{t_i} \lambda^*(s) ds\Big)\Big] \exp\Big(-\int_{t_n}^{T} \lambda^*(s) ds\Big) \\ &= \Big[\prod_{i=1}^{n} \lambda^*(t_i)\Big] \exp\Big(-\int_0^T \lambda^*(s) ds\Big)\end{aligned} \quad (4)$$

3) Checking the goodness-of-fit: Residual analysis [1]

Let's consider a sequence of occurrence times $\{t_1, t_2, ...\}$ and a monotonic, continuous compensator $\Lambda(.)$ such that $\lim_{t \to +\infty} \Lambda(t) = +\infty$ almost surely and $\Lambda(t) = \int_0^t \lambda^*(s) ds$ (Λ is called compensator). The sequence $\{\Lambda(t_1), \Lambda(t_2), ...\}$ is a Poisson process with an unit rate if and only if $\{t_1, t_2, ...\}$ is a realisation from the point process defined by $\Lambda(.)$.

$$\begin{aligned}\lambda'(t) &= -\frac{d}{dt}[\log(1 - F'(t))] = -\frac{d}{dt}[\log(1 - F[\Lambda^{-1}(t)])] \\ &= \frac{1}{1 - F[\Lambda^{-1}(t)]} \frac{d}{dt}[F[\Lambda^{-1}(t)]] = \frac{f[\Lambda^{-1}(t)]}{1 - F[\Lambda^{-1}(t)]} \frac{d}{dt}[\Lambda^{-1}(t)] \\ &= \lambda[\Lambda^{-1}(t)] \frac{d}{dt}[\Lambda^{-1}(t)] = \frac{d\Lambda}{dt}[\Lambda^{-1}(t)] \frac{d}{dt}[\Lambda^{-1}(t)] = \frac{d}{dt}[\Lambda(\Lambda^{-1}(t))] = \frac{dt}{dt} = 1\end{aligned}$$

Preliminary example

We first applied Hawkes process to estimate the likelihood of a **vehicle change**, in order to be integrated into a recommender system of new auto insurance covers.
We tested the most current choice for excitation function: $\mu_{j,i}(t) = \alpha \exp(-\beta t)$

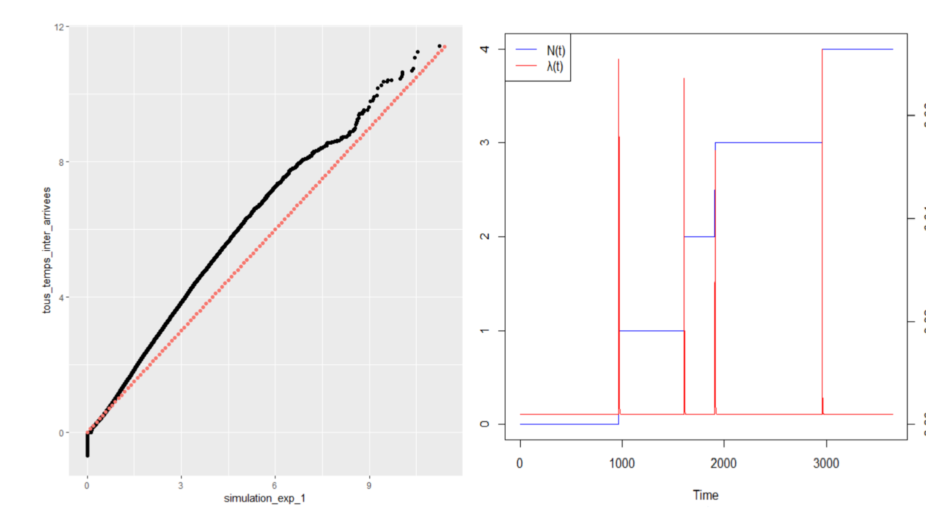

Fig. 2: QQ-plot comparing transformed process with compensator and Poisson process of rate 1 / Simulation of estimated Hawkes process over 10 years

In a first test on 100 customers, our recommender system had a 30% acceptance rate.

Future developments

- **Model every kind of events listed**: build a multivariate Hawkes process taking into account every event.
- **Select more accurate excitation functions**: simple functions such that exponential are not sufficient to describe the influence of an occurrence. More complex functions could allow a better modelling.
- **Introduce external information into background intensity**: instead of considering a constant background intensity, making it dependant on external information (localization, customer's profile, etc.) could make it more accurate.

References

[1] D. Vere-Jones D. Daley. *An Introduction to the Theory of Point Processes: Volume I: Elementary Theory and Methods*. Vol. 1. Springer, 2003.

[2] Alan G. Hawkes. "Spectra of some self-exciting and mutually exciting point processes". In: *Biometrika* 58 (1971), pp. 83–90.

[3] Philip K. Pollett Patrick J. Laub Thomas Taimre. "Hawkes Processes". In: *arXiv preprint arXiv:1507.02822* (2015).

[4] Jakob Gulddahl Rasmussen. "Lecture Notes: Temporal Point Processes and the Conditional Intensity Function". In: *arXiv:1806.00221v* (2018).